INTERACTION OF A CYLINDRICAL SHOCK WITH A THIN-WALLED PERFORATED SCREEN

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The non-self-similar problem of cylindrical shock interaction with a coaxial thin-walled perforated screen of cylindrical shape is investigated within the framework of modeling the screen by a discontinuous surface. The solution is constructed by the S. K. Godunov numerical method. In addition to the general relationships on the discontinuity, a necessary number of additional boundary conditions is invoked that express the specifics of the local gas progress through the perforation. Exactly as in known examples [1-3] of the construction of additional boundary conditions, an assumption about the quasistationary nature of the local flow is used. The suitability of such boundary conditions for the analysis of non-stationary processes requires special confirmation. A comparison of the results of solving the self-similar problem of the normal passage of a plane shock through a plane penetrable wall with experiment is made in [3]. The comparison of non-self-similar solutions obtained under conditions of a time-varying intensity of the discontinuity modeling the perforated screen with experiment is of interest. In this connection, we note the experimental data obtained in [4] and the results of the numerical investigation in [5].

1. Let us examine the problem of destruction of a gas-filled cylindrical shell of radius r_0 within a coaxial cylindrical, uniformly perforated, stiff screen of radius R ($R > r_0$). Initially, the gas is everywhere at rest, the pressure density within the shell are constant and equal p_*, ρ_* and outside are p_0, ρ_0. The motion starts at the time $t = 0$ with the decay of the discontinuity originating because of instantaneous "destruction" of the shell. The screen is modeled by the surface of discontinuity of ideal perfect gas parameters. The wave motion of the medium for $t > 0$ is described by a system of one-dimensional nonstationary Euler equations [6]

$$\frac{\partial}{\partial t}\begin{pmatrix} r\rho \\ r\rho u \\ r\rho\left(\frac{u^2}{2}+\frac{p}{(\gamma-1)\rho}\right)\end{pmatrix}+\frac{\partial}{\partial r}\begin{pmatrix} r\rho u \\ r(p+\rho u^2) \\ r\rho u\left(\frac{u^2}{2}+\frac{c^2}{\gamma-1}\right)\end{pmatrix}=\begin{pmatrix}0\\ p\\ 0\end{pmatrix}. \tag{1.1}$$

We have the boundary condition of nonpenetration $u = 0$ on the axis $r = 0$. In addition to the fundamental boundary conditions resulting from the general integral conservation laws on the discontinuity [1-3]

$$[\rho u]=0,\ [p+\rho u^2]=-F,\left[\frac{u^2}{2}+\frac{c^2}{\gamma-1}\right]=0, \tag{1.2}$$

on the surface of the discontinuity $r = R$ additional relationships must be relied upon which will reflect the specifics of the local gas flow through the perforations.

It is assumed in (1.1) and (1.2) that r is the distance to the axis of symmetry, u is the radial velocity, γ is the adiabatic index, $c = (\gamma p/\rho)^{1/2}$ is the sound speed, F is the force acting from the gas per unit screen area, the square brackets denote the jump $[Q] = Q_2 - Q_1$ for any parameter Q, and here and later the subscripts 1 and 2 will separate the gas parameters in the windward and leeward sides of the discontinuity, respectively.

Let us introduce the Mach number $M = |u/c|$. The total quantity of additional boundary relationships to system (1.1) and (1.2) is determined by the condition of evolution of the discontinuity [7] in each of the following four possible flow modes: for $M_1 < 1$, $M_2 < 1$ one additional relationship is required, for $M_1 < 1$, $M_2 \geq 1$ two, for $M_1 \geq 1$, $M_2 \geq 1$ one, and

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$M_1 \geq 1$, $M_2 < 1$ no additional relationships are required (let us note that in the terminology of [3] the second of the cases mentioned corresponds to a mode with so-called "closed separation zones").

As in [3, 8], specific expressions are obtained for the additional boundary conditions from an examination of the stationary local gas flow schemes through the perforations. Distinctions are due to the differences between the geometric $\sigma = S_0/S$ and the effective $\varepsilon = S_m/S$ penetrability factors that hold for thin-walled screens ($h \ll d$). Here h and d are the characteristic values of the longitudinal and transverse dimensions of the perforation channels, S is the area of a given screen section, S_0 is the area of minimal through section of perforations on the section S, and S_m is the area of the minimal section of gas jets issuing from the perforation cells.

According to data in [8, 9], the quantity ε for $h \ll d$ and $M_1 < 1$, $M_2 < 1$ depends substantially or not only σ but also the penetrating gas parameters even after attainment of the sound speed in the section S_m. Using the asymptotics obtained in [8, 9], for the relationships between the parameters of the local gas flow through the holes and slits with sharp edges, the following approximate expression can be constructed for the additional boundary condition for thin-walled perforated screens for $M_1 < 1$, $M_2 < 1$:

$$C_p = \frac{p_1 - p_2}{\frac{1}{2}\rho_1 u_1^2} = k(\gamma, \sigma, M_1)\,\zeta(\sigma), \tag{1.3}$$

where ζ is the local drag coefficient of the perforations as $M_1 \to 0$ taken according to the hydraulic formula [10], $\zeta = \left(1 + \sqrt{\frac{1-\sigma}{2}} - \sigma\right)^2 \Big/ \sigma^2$, and the dependence of k on M_1, σ, γ is given by the formulas

$$k = Z + \frac{\gamma - 1}{\gamma + 1}\,\frac{ZL^2}{1 - \frac{\gamma}{\gamma+1}\zeta Z L^2}, \quad L^2 = \frac{(\gamma+1)M_1^2}{2 + (\gamma - 1)M_1^2}, \tag{1.4}$$

$$Z = (1 - \delta L^2 N^{-2})^{-\frac{1}{4}}, \quad \delta = 1 - \left(\frac{\gamma}{\gamma+1}\,\frac{\zeta N^2}{1-N}\right)^4,$$

$$0 < L < N < 1, \quad (\gamma+1)N^{\gamma-1} - (\gamma-1)N^{\gamma+1} = 2\sigma^{\gamma-1}.$$

The quantity k has the meaning of the so-called "compressibility correction" in the hydraulic relationship $C_p = \zeta(\sigma)$ [10]. For air this correction is known from experiment. Comparison of the experimental points in [10] with the curves (1.4), which are constructed for $\gamma = 1.4$ and different σ, is shown in Fig. 1 (The steep sections of the curves correspond to flow "suppression" in the section S_m). The simple relationship

$$\frac{\rho_2}{\rho_1} = 1 - \frac{\gamma}{\gamma+1}\zeta Z L^2 \tag{1.5}$$

results from (1.2)-(1.4) and is convenient for analysis of the system of boundary conditions, for instance, to check out the inequality $[p\rho^{-\gamma}] \geq 0$, expressing the condition of a nondecrease in the entropy on the discontinuity under consideration. This comparison with experiment is the foundation for using (1.3) (or (1.5)) as the additional boundary condition in the fundamental mode $M_1 < 1$, $M_2 < 1$.

2. Solution of the problem posed was constructed numerically by the finite-difference method of S. K. Godunov [6] on a uniform fixed computation mesh. The arithmetic average of the pressures at the upper and lower time layers was taken for the approximation of the nondivergent term p in the motion equations (1.1). Within the screen, 20 computational cells were arranged, and an increase in this number did not result in any noticeable change in the results. The time spacing was determined by the stability conditions [6].

All the parameters represented below should be understood as dimensionless: linear dimensions are referred to r_0, the density to ρ_0 the pressure and load on the screen to p_0, and the time to $r_0\rho_0^{1/2}p_0^{-1/2}$. The numerical computation of the problem was executed for $\gamma = 1.4$ and several values of the parameters p_*, σ, R in the ranges $1.3 \leqslant p_* \leqslant 10$; $0.02 \leqslant \sigma \leqslant 1$; $2 \leqslant R \leqslant 4$.

As a result of dissociation of the initial discontinuity, a divergent shock occurs at t = 0 on the line r = 1. At the time when its leading front reaches the line r = R, interaction

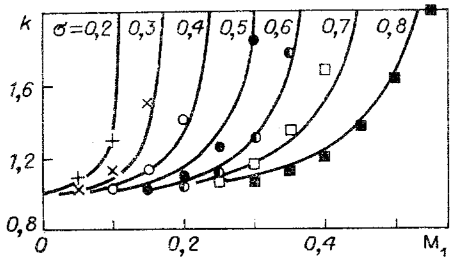

Fig. 1

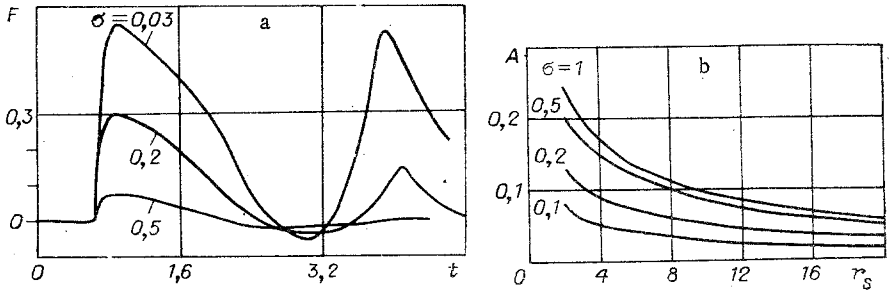

Fig. 2

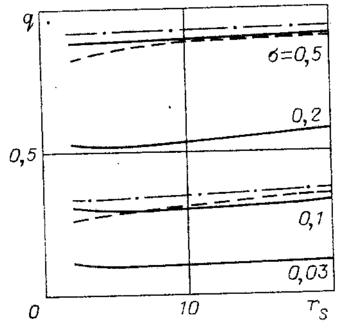

Fig. 3

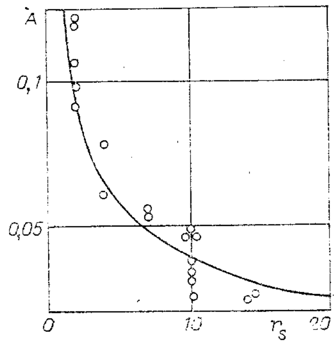

Fig. 4

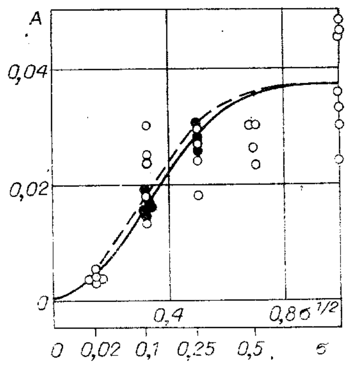

Fig. 5

with the screen starts: a reflected shock appears that moves to the axis of symmetry, and the initial wave weakened by the screen that passes outside and whose front $r = r_s(t)$ starts to be propagated into surround space over the unperturbed gas $p = \rho = 1$. At this time the screen experiences shock loading.

Results of computations for $p_* = \rho_* = 2$, $R = 2$ are shown in Fig. 2 for different values of the penetration factor σ. The oscillatory nature of the nonstationary load $F = F(t)$ acting on the screen (Fig. 2a) is caused by repeated interaction between the screen and inwardly reflected waves. The degree of penetrability substantially affects the relationship between the amplitudes of the first and repeated loading phases of the screen. One of the fundamental characteristics of the passing wave is the pressure drop $A = p(r_s) - 1$ on its front $r = r_s(t)$. The dependence of A on r_s and σ is monotonic (Fig. 2b).

The ratio $A(\sigma)/A(1)$ characterizes the degree of weakening of the first passing wave by the screen. The dependence of q on r_s and σ is constructed in Fig. 3 for the case R = 2 and different $p_* = \rho_* = b$ (b = 2 are the solid lines, b = 1.6 the dash—dot, and b = 10 the dashes).

The results of the present computations are compared with the experimental data in [4] in Figs. 4 and 5. In the experiment the shell was filled with compressed air to $p_* = 1.6$; however, a weaker shock was obtained than for ideal dissociation of the discontinuity for the mentioned value of p_*, as is explained by expenditure of parts of the compressed air energy in scattering of the material components of the shell. The dependence, observed in experiment [4], of the pressure drop A at the shock front on the distance r_s to the axis of symmetry is shown by points in Fig. 4 (there is no screen, $\sigma = 1$). This dependence is described well by the solution of the problem of dissipation of a cylindrical discontinuity if $p_* = 1.35$ and $\rho_* = 1.6$ are taken (curve in Fig. 4).

The computed and experimental versions of the dependence of A on the screen penetrability σ are shown for $r_s = 10$ in Fig. 5 for two cases of the screen arrangement with respect to the shell: the solid line and open points correspond to R = 2, while the intermittent line and dark points correspond to R = 4. The experimental data presented have been obtained in [4] for uniformally perforated circular holes in thin-walled metal screens with h/d = 0.1, σ = 0.02, 0.1, 0.25, 0.5. Agreement between the theoretical and experimental results indicates the possibility of using boundary conditions based on stationary schemes of local gas flow through perforations, to analyze nonstationary processes.

3. An analogous problem, but with another formulation, was considered in [5]. The perforation of the cylindrical screen consisted of six longitudinal slits, the ratio between the perforation spacing and the screen radius was $\pi/3$, i.e., the scale of the "local" flow through the perforation was comparable to the screen dimension. Under these conditions a quantitative comparison between the present computations and the results in [5] is not legitimate since the meaning of the concept of a surface of discontinuity as modelling the perforated screen is lost here. Nevertheless, on the basis of the results in [5], a number of deductions can be made in favor of the problem formulation we used. It turns out that even at such small distances as two perforation spacings and appropriate short times, substantial equilibrium of the field of gas flow parameters occurs in the angular coordinate. This indicates the expediency of a one-dimensional approach. And even more, than for the analysis of another perforation, in the form of circular holes, say, as in [4], three-dimensional nonstationary equations would have to be solved by the method of [5], while it is possible to limit oneself to the solution of the one-dimensional problem.

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